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CRM: Prior means and prior medians

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In the CRM and BMA-CRM methods, as implemented at M. D. Anderson Cancer Center, the probability of toxicity at the ith dose is modeled as

$$p_i^{\exp(\alpha)}$$

where $0 < p_i < 1$ and α is normally distributed *a priori* with mean 0 and variance $\sigma^2 = 2$, though the specific value of σ^2 is not very important. See [1] and [2].

There has been some discussion about the role of the p's, specifically whether software should prompt a statistician for the values of the p's directly, or whether the software should let the statistician enter the prior mean toxicities and solve for the corresponding p's. This note explains how these two approaches are related.

Let α be as described above. Observe that

$$\begin{aligned} P(p^{\exp(\alpha)} < x) &= P\left(\alpha > \log\left(\frac{\log x}{\log p}\right)\right) \\ &= \Phi^c\left(\log\left(\frac{\log x}{\log p}\right)/\sigma\right) \end{aligned}$$

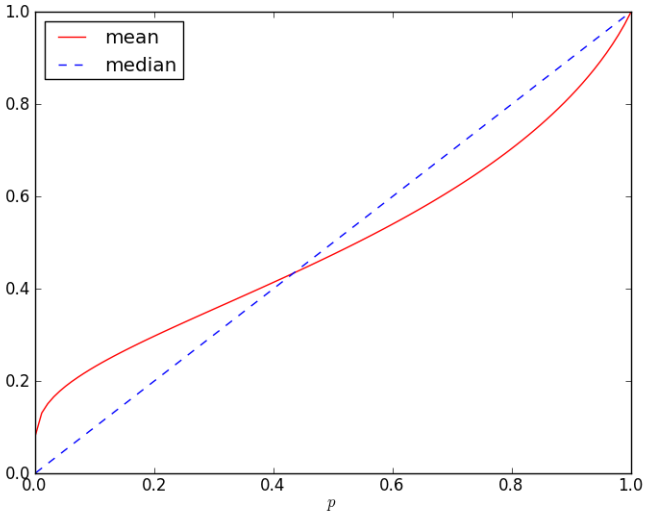
where Φ^c is the complementary CDF of a standard normal distribution.

Note that by setting $x = p$,

$$P(p^{\exp(\alpha)} < p) = \Phi^c(0) = 1/2$$

and so p is the median of $Y = p^{\exp(\alpha)}$. Therefore the discussion of whether to specify p or the prior mean of Y could be cast as *whether to specify the mean or median of* Y.

The following plot shows that the prior mean and median are fairly similar for moderate probabilities p. For small values of p, however, the mean is relatively much larger than the median. This means that small median values lead, perhaps unexpectedly, to much larger prior means. Conversely, small means lead to extremely small medians.



The following tables give a few corresponding values of mean and median.

median	mean
0.05	0.1875
0.10	0.2308
0.15	0.2658
0.20	0.2972
0.25	0.3269
0.30	0.3559
0.35	0.3847
0.40	0.4138
0.45	0.4435
0.50	0.4742
0.55	0.5062
0.60	0.5400
0.65	0.5760
0.70	0.6147
0.75	0.6570
0.80	0.7039
0.85	0.7571
0.90	0.8191
0.95	0.8951

mean	median
0.05	1.89e-05
0.10	0.0028
0.15	0.0207
0.20	0.0628
0.25	0.1266
0.30	0.2047
0.35	0.2898
0.40	0.3765
0.45	0.4608
0.50	0.5405
0.55	0.6142
0.60	0.6815
0.65	0.7420
0.70	0.7960
0.75	0.8437
0.80	0.8855
0.85	0.9218
0.90	0.9528
0.95	0.9789

References

[1] CRM Simulator software available at `http://tinyurl.com/CRMSimulator`

[2] BMA-CRM software available at `http://tinyurl.com/BMA-CRM`