Relating Φ and erf

There's nothing profound here, just simple but error-prone calculations that I've done so often that I decided to save the results.

Let $\Phi_{\mu,\sigma}(x)$ be the CDF of a normal random variable with mean μ and standard deviation σ.

$$\Phi_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right) dt$$

Let $\Phi(x)$ with no subscripts be the CDF of a standard normal random variable, *i.e.* $\mu = 0$ and $\sigma = 1$. Let $\Phi_c(x) = 1 - \Phi(x)$, the complementary CDF of a standard normal.

The error function is defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, dt$$

and the complementary error function is defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)\, dt = 1 - \operatorname{erfc}(x).$$

These relations below follow directly from the definitions.

$$\begin{aligned}
\Phi_{\mu,\sigma}(x) &= \Phi\left(\frac{x-\mu}{\sigma}\right) \\
\Phi(x) &= \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right) \\
\operatorname{erf}(x) &= 2\Phi(\sqrt{2}x) - 1 \\
\Phi^{-1}(x) &= \sqrt{2}\operatorname{erf}^{-1}(2x-1) \\
\operatorname{erf}^{-1}(x) &= \frac{1}{\sqrt{2}}\Phi^{-1}\left(\frac{x+1}{2}\right) \\
\Phi_{\mu,\sigma}(x) &= \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right) \\
\Phi_{\mu,\sigma}^{-1}(x) &= \sqrt{2}\sigma\operatorname{erf}^{-1}(2x-1) + \mu \\
\Phi_c(x) &= \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \\
\Phi_c^{-1}(x) &= \Phi^{-1}(1-x) \\
\Phi_c^{-1}(x) &= \sqrt{2}\operatorname{erfc}^{-1}(2x) \\
\operatorname{erfc}(x) &= 2\Phi_c(\sqrt{2}x) \\
\operatorname{erfc}^{-1}(x) &= \frac{1}{\sqrt{2}}\Phi_c^{-1}\left(\frac{x}{2}\right)
\end{aligned}$$