Define $T(n,1) = n$ and for $k > 0$ define

$$T(n,k) = \sum_{i=1}^{n} T(i,k-1).$$

The numbers $T(n,2)$ are the triangle numbers, $T(n,3)$ are the tetrahedral numbers, and $T(n,k)$ are the k-dimensional analogs of the tetrahedral numbers.

Theorem:

$$T(n,k) = \binom{n+k-1}{k}.$$

Proof:

Since $T(1,k) = 1$ and $T(n,1) = n$, the theorem holds when $n < 2$ or $k < 2$. Now assume the theorem holds for all $n < m$ and $k < m$.

$$\begin{aligned}
T(n,m) &= \sum_{i=1}^{n} T(n,m-1) \\
&= T(n,m-1) + \sum_{i=1}^{n-1} T(n,m-1) \\
&= T(n,m-1) + T(n-1,m) \\
&= \binom{n+m-2}{m-1} + \binom{n+m-2}{m} \\
&= \binom{n+m-1}{m}
\end{aligned}$$

The last line follows from the binomial coefficient identity

$$\binom{a}{b} = \binom{a-1}{b} + \binom{a-1}{b-1}.$$