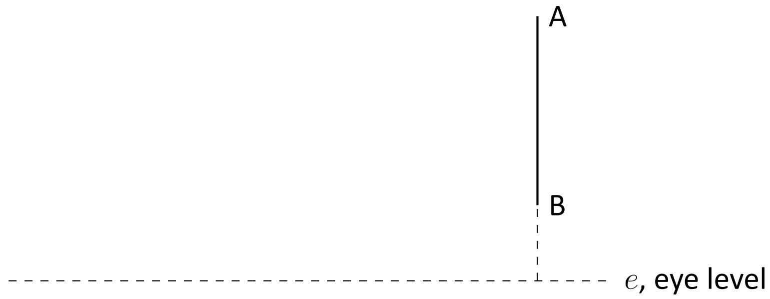
1 En Old Problem of Müller

In 1471 Johannes Müller wrote: Where you should stand so that a vertical bar appears longest?

Here is a try to solve the problem with pure geometrical reasoning.

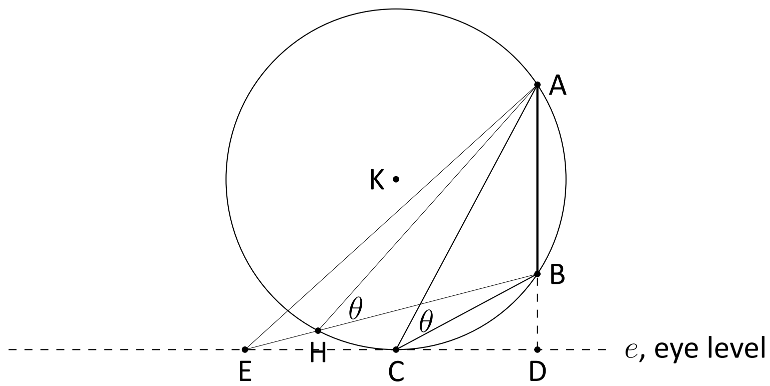
Lets think that there is a vertical bar of height h, starting at height b and ending at height a over the eye level (line e), as in the graph.



The desired point (C) is the point that the circle that passes through the points A and B is touching tangently the line e of eye level.

The graph below is proving the proposition. Consider another point E in e and the lines EA and EB. Let H be the common point of EB and the circle. Then $\text{B}\hat{\text{H}}\text{A} = \text{B}\hat{\text{C}}\text{A} = \theta$ and in the triangle HEA

$$\theta = \text{H}\hat{\text{E}}\text{A} + \text{H}\hat{\text{A}}\text{E} \Rightarrow \theta > \text{H}\hat{\text{E}}\text{A}$$



So θ is the desired max angle.

We have also from a known theorem of elementary geometry that from point D:

$$DA \cdot DB = DC^2$$

so

$$ab = x^2$$

or

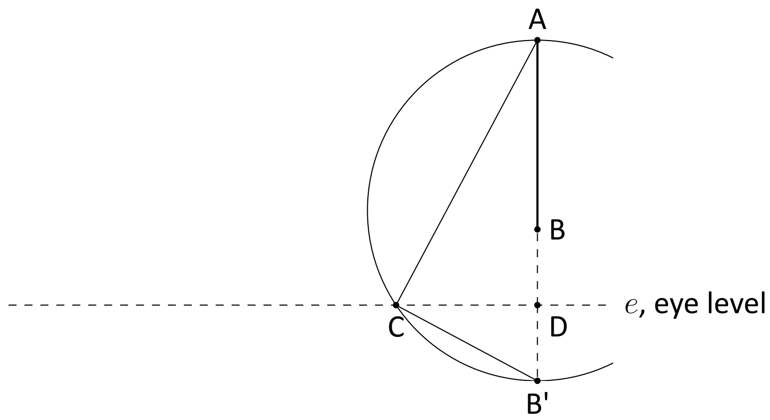
$$x = \sqrt{ab}$$

And finaly the crucible question: *How do we find the point C, or the circle (K, KA)?*

This also is answered in elementary geometry. We can find the μέσο ανάλογο (geometric mean) x of any given lenghts a and b. Lets construct it *with compass and straightedge*.

First we find the symmetrical point B' of B. Now we have the two given lenghts a and b, side by side. Then we find the middle M of B'A, and we make the circle (M, MA). C is the intersection point of the eye level line e and the circle (K, KA), and is the desired point because in the right angled triangle CAB', $CD^2 = B'D \cdot DA$ or $CD^2 = DB \cdot DA$.

The graph below shows the construction.



So in 1471 they have not invented yet calculus but they knew much of geometry!

Best wishes

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P.S. Forgive my many errors. I dont have english spell checking in TeXnicCenter!