

Let f and g be two (not necessarily distinct) Airy functions. We want to show that their product satisfies the differential equation

$$y''' - 4xy' - 2y = 0. \quad (1)$$

Since f and g are Airy functions, they satisfy the differential equation

$$y'' - xy = 0. \quad (2)$$

We set $y = fg$ and compute the left side of equation (1), taking advantage of the fact that f and g satisfy equation (2).

$$\begin{aligned} (fg)''' - 4x(fg)' - 2fg &= f'''g + 3f''g' + 3f'g'' + fg''' - 4xf'g - 4xfg' - 2fg \\ &= (xf)'g + 3xfg' + 3xf'g + f(xg)' - 4xf'g - 4xfg' - 2fg \\ &= fg + xf'g + 3xfg' + fg + xfg' - 4xf'g - 4xfg' - 2fg \\ &= 0 \end{aligned}$$