The distance between a planet in an elliptical orbit and its star is given by

$$r = a(1 - e\cos E)$$

where a is the semimajor axis, e is eccentricity, and E is the eccentric anomaly.

Eccentric anomaly is a geometric way of describing a planet's position in its orbit. What we care about is the mean anomaly M, which is based on time rather than geometry:

$$M = \frac{2\pi t}{T}$$

where T is the period of the orbit.

The relation between M and E is given by Kepler's equation

$$M = E - e\sin E.$$

The average distance to the star is then

$$\begin{aligned}
\frac{1}{T}\int_0^T r\,dt &= \frac{1}{T}\int_0^{2\pi} a(1 - e\cos E)\,dt \\
&= \frac{1}{T}\int_0^{2\pi} a(1 - e\cos E)\,d\left(\frac{TM}{2\pi}\right) \\
&= \frac{1}{T}\int_0^{2\pi} a(1 - e\cos E)\cdot\frac{T}{2\pi}\,d\left(E - e\sin E\right) \\
&= \frac{1}{T}\int_0^{2\pi} a(1 - e\cos E)\cdot\frac{T}{2\pi}(1 - e\cos E)\,dE \\
&= \frac{a}{2\pi}\int_0^{2\pi} (1 - e\cos E)^2\,dE \\
&= a\left(1 + \frac{e^2}{2}\right)
\end{aligned}$$